Entry Level Literacy and Numeracy Assessment for the Electrotechnology Trades

Enrichment Resource

UNIT 14: Trigonometry
TRIGONOMETRY

Trigonometry is a means of solving for sides and angles in a right-angled triangle. This information can then be used to calculate the magnitude and direction of forces in the electrical trade.

For example, in circuits which contain inductance and capacitance trigonometry can be used to determine the relationship between the current and voltage. The functions are also used to calculate phase angles.

![Diagram of a right-angled triangle with sides Z, X, and R and angle θ.]

LEARNING OUTCOME

- Can use the trigonometric functions to calculate an unknown side/s and angle/s in a right-angled triangle.

PERFORMANCE CRITERIA

- Uses the scientific calculator to find the sine, cosine or tangent of an angle.
- Identifies when to use the sin, cos or tan ratio to find an unknown side or angle in a right-angled triangle.
- Transposes the trigonometric ratios in order to solve for sides or angles.
- Uses the scientific calculator to solve worded problems involving the trigonometric functions.
TRIGONOMETRIC FUNCTIONS

Trigonometric functions provide information about the size of the angles in right angled triangles. These functions express the ratio between any two sides of a right triangle.

When an angle is given (the Greek letter $\theta$ is used as a general angle) then the sides of the triangle are labelled with respect to this angle ($\theta$).

1. The longest side is the hypotenuse.
2. The side opposite the angle is the opposite side.
3. The side next to the angle is the adjacent side.
3 functions expressing a ratio of the length of one side to another:

1. Sine of $\theta$ = \[
\frac{\text{opposite side}}{\text{hypotenuse}}
\]
or
\[
\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{X}{Z}
\]

2. Cosine of $\theta$ = \[
\frac{\text{adjacent side}}{\text{hypotenuse}}
\]
or
\[
\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{R}{Z}
\]

3. Tangent of $\theta$ = \[
\frac{\text{opposite side}}{\text{adjacent side}}
\]
or
\[
\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{X}{R}
\]
TRIGONOMETRIC FUNCTIONS – USING THE CALCULATOR

A scientific calculator can be used to obtain the sine, cosine or tangent of an angle. The answer is expressed in radians.

Note: The following steps are used on a Sharp calculator, other calculators may use different steps. Read your operators manual to determine the steps used on your calculator.

ANGLES LESS THAN 90°

Example 1

Using the Microsoft calculator:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Keysteps</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin 18°</td>
<td>1 8 sin</td>
<td>0.3090</td>
</tr>
<tr>
<td>cos 40°</td>
<td>4 0 cos</td>
<td>0.7660</td>
</tr>
<tr>
<td>tan 70°</td>
<td>7 0 tan</td>
<td>2.7474</td>
</tr>
<tr>
<td>cos 75.8</td>
<td>7 5 . 8 cos</td>
<td>0.2453</td>
</tr>
<tr>
<td>sin 14.2</td>
<td>1 4 . 2 sin</td>
<td>0.2453</td>
</tr>
</tbody>
</table>
Example 2

Finding the sine, cosine and tangent of angles expressed in degrees and minutes using the D°M’S key (degrees, minutes, seconds) of some calculators:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Keysteps</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>cos 26°54’</td>
<td>cos 2 6 D°M’S 5 4 =</td>
<td>0.8918</td>
</tr>
<tr>
<td>sin 60°12’</td>
<td>sin 6 0 D°M’S 1 2 =</td>
<td>0.8678</td>
</tr>
</tbody>
</table>

EXERCISE 1

Use the calculator to find the values of the following. Give answers correct to 4 decimal places.

a. sin 30°

b. cos 65°

c. tan 20°

d. sin 71°13’

e. cos 8°19’

f. tan 23.5°

g. sin 79° 3’

h. cos 81°45’

Use the answer sheet to check your work
FINDING THE UNKNOWN SIDES

To use the trigonometric ratios to find unknown sides, it is important to be clear about which side is the opposite, adjacent and hypotenuse in any given triangle.

EXERCISE 2

Label the sides of each of the triangles below to show which sides are the opposite, adjacent or hypotenuse.

a. 

b. 

c. 

d. 
Example 3

Use the sin ratio to find the side marked x.

Remember: \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \)

\[
\sin 60^\circ = \frac{x}{30}
\]

\[
. \ . \ x = 30 \times \sin 60^\circ
\]

\[
= 30 \times 0.8660
\]

\[
= 25.9808
\]

\[
= 25.98
\]

Using the Microsoft calculator:

3 0 x 6 0 sin

Answer: 25.9808
Rounded: 25.98
EXERCISE 3

Use the sin ratio to find the side marked X.

a.

\[
\sin 70^\circ = \frac{X}{45} \\
\therefore X = 45 \times \sin 70^\circ
\]

b.

\[
\sin 51^\circ 55' = \frac{X}{380}
\]

\[
X = 380 \times \sin 51^\circ 55'
\]

c.

\[
\sin 44^\circ 5' = \frac{X}{720}
\]

\[
X = 720 \times \sin 44^\circ 5'
\]
Example 4

Use the cos ratio to find the side marked R. Give answer correct to 2 decimal places.

Remember: \( \cos \theta = \frac{\text{adj}}{\text{hyp}} \)

\[
\cos 55^\circ = \frac{R}{50} \\
R = 50 \times \cos 55^\circ \\
= 50 \times 0.5736 \\
= 28.6788 \\
\approx 28.68
\]

Using the calculator

```
5 0 x 5 5 cos
```

Answer: 28.6788
Rounded: 28.68
EXERCISE 4

Use the cos ratio to find the side marked R. Give answers correct to 2 decimal places.

a.

\[
\begin{align*}
\text{R} & \quad 38^\circ \\
95 & \\
\end{align*}
\]

b.

\[
\begin{align*}
\text{R} & \quad 59.12^\circ \\
310 & \\
\end{align*}
\]

c.

\[
\begin{align*}
\text{R} & \quad 28.5^\circ \\
208 & \\
\end{align*}
\]
Example 4

Use the tan ratio to find the side marked Z.

\[
\tan 38° = \frac{Z}{410} \\
\therefore Z = 410 \times \tan 38° \\
= 410 \times 0.7813 \\
= 320.3271 \\
= 320.33
\]

Using the calculator

```
4 1 0 x 3 8 tan
```

Answer: 320.271
Rounded: 320.33
EXERCISE 5

Use the tan ratio to find the side marked Z.

a.

b.

Use the answer sheet to check your work.
EXERCISE 6

Find the unknown sides (W) in the following triangles. You will need to decide which ratio to use for each of the triangles.

ie. \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \text{or} \quad \tan \theta = \frac{\text{opp}}{\text{adj}} \)

a.

\[ \begin{array}{c}
\text{W} \\
31.5
\end{array} \]

\[ \begin{array}{c}
\text{28°}
\end{array} \]

b.

\[ \begin{array}{c}
\text{W} \\
49.21
\end{array} \]

\[ \begin{array}{c}
\text{49°}
\end{array} \]

c.

\[ \begin{array}{c}
\text{W} \\
480
\end{array} \]

\[ \begin{array}{c}
\text{22°}
\end{array} \]

✓ Use the answer sheet to check your work.
TRANSPOSING THE RATIOS

Example 5

Find the unknown side \( Z \).

\[
\sin 34^\circ = \frac{240}{Z} \quad \text{(Multiply both sides by } Z) \\
Z \times \sin 34^\circ = 240 \\
Z = \frac{240}{\sin 34^\circ} \\
Z = \frac{240}{0.5592} \\
\therefore Z = 429.19
\]

Using the calculator

\[
\begin{align*}
2 & \quad 4 & \quad 0 & \quad - & \quad 3 & \quad 4 & \quad \text{sin} & \quad = \quad \text{Answer: 429.19}
\end{align*}
\]
EXERCISE 7

Find the unknown sides (W) in the following triangles. Give answers correct to 2 decimal places.

a.

\[
\begin{align*}
\text{19.21} & \quad \text{W} \\
68^\circ 5' & \quad \text{68\textdegree 5'}
\end{align*}
\]

b.

\[
\begin{align*}
\text{20.67} & \quad \text{W} \\
50^\circ 8' & \quad \text{50\textdegree 8'}
\end{align*}
\]

c.

\[
\begin{align*}
74 & \quad \text{W} \\
29^\circ 40' & \quad \text{29\textdegree 40'}
\end{align*}
\]

d.

\[
\begin{align*}
37.36 & \quad \text{W} \\
37^\circ 49' & \quad \text{37\textdegree 49'}
\end{align*}
\]

e.

\[
\begin{align*}
198.5 & \quad \text{W} \\
42^\circ & \quad \text{42\textdegree}
\end{align*}
\]

Use the answer sheet to check your work.
FINDING ANGLES

The three trigonometric ratios can be used to find an angle when two or three sides in the triangle are known.

The first step in such a calculation is to determine which ratio needs to be used. Then the inverse keys on the scientific calculator can be used to calculate the size of the angle.

The inverse trigonometric keys (or arc functions) are:

\[ \sin^{-1}, \cos^{-1}, \tan^{-1} \]

eg. \[ \sin^{-1} 0.45 \text{ means "the sine who's angle is 0.45"} \]
\[ \cos^{-1} 0.6 \text{ means "the cosine who's angle is 0.6"} \]

Example 6

Find the angle \( \theta \)

![Diagram of a right triangle with sides 3, 5, and \( \theta \).]

The opposite side and hypotenuse are given \( \therefore \) the sin ratio can be used to find \( \theta \).

\[
\sin \theta = \frac{\text{opposite}}{\text{Hypotenuse}}
\]

\[ \sin \theta = \frac{3}{5} \]

\[ = 0.6 \]

\[ \sin^{-1} 0.6 = 36°52' \]

\[ 3 + 5 = 2^{nd} \text{ F } \sin^{-1} = \text{D°M’S} \text{ Answer: } 36°52' \]
EXERCISE 8

Use the arc functions (inverse trig) keys to find the following. Express the answers in degrees and minutes.

a. \( \sin^{-1} 0.7705 \)

b. \( \sin^{-1} 1 \)

c. \( \cos^{-1} 1 \)

d. \( \cos^{-1} 0.9655 \)

e. \( \tan^{-1} 0.2793 \)

f. \( \cos^{-1} 0.2924 \)
EXERCISE 9

Find the unknown angle $\theta$ in the following triangles. You will need to first of all decide which ratio to use and then use the appropriate inverse key.

a. (Opposite and hypotenuse are known $\therefore$ use $\sin^{-1}$)

b. 

c. 

d. 

e. In a right angled triangle $\theta = 54^\circ$ and the hypotenuse is 10mm in length. Find the lengths of the opposite and adjacent sides.

f. Angle $\theta$ in a right angled triangle is $24^\circ$ and the opposite side is 13 units in length. Solve for the lengths of the other two sides.
EXERCISE 10

a. The diagram below represents the cross-section of a roof of a house. If the pitch of the roof is 9°, how high is the top of the roof above the ceiling?

b. An electrician needs to run some conduit from the base of a tall building to the top. He uses the measurements shown below to calculate the required length of conduit. How much conduit is needed?

c. Two cables are used to secure a TV antenna 19.5 metres high. Calculate the length of each cable given the angles shown in the diagram.
d. Angle detecting switches for burglar alarms are rated to activate at various angles. If a 1200mm long window opens 300mm at the base, specify the angle of the switch you would need.

**ANGLES GREATER THAN 90°**

The value of the sin, cos and tan ratios are either negative or positive depending on the size of the angle. The diagram below summarises the signs of the trigonometric functions.

<table>
<thead>
<tr>
<th>Quadrant 2</th>
<th>Quadrant 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin+</td>
<td>sin+</td>
</tr>
<tr>
<td>cos-</td>
<td>cos+</td>
</tr>
<tr>
<td>tan-</td>
<td>tan-</td>
</tr>
<tr>
<td>Quadrant 3</td>
<td>Quadrant 4</td>
</tr>
<tr>
<td>sin-</td>
<td>sin-</td>
</tr>
<tr>
<td>cos-</td>
<td>cos+</td>
</tr>
<tr>
<td>tan+</td>
<td>tan-</td>
</tr>
</tbody>
</table>

The features of the sine and cosine function are shown below highlighting the relationship between angle size and positive and negative values.

![Graphs of sine and cosine functions](image)

(a) the sine function  
(b) the cosine function

The sine and cosine functions are the main functions used when studying AC voltages and currents.
EXERCISE 11

Use the calculator to compare the values of the following functions.

a. \( \sin 30^\circ \)
b. \( \sin 210^\circ \)
c. \( \cos 184^\circ \)
d. \( \cos 4^\circ \)
e. \( \sin 97^\circ \)
f. \( \sin 83^\circ \)
g. \( \cos 20^\circ \)
h. \( \cos 340^\circ \)
i. \( \sin 330^\circ \)
j. \( \cos 300^\circ \)
ANSWERS

EXERCISE 1

a. 0.5000  
b. 0.4226  
c. 0.3640  
d. 0.9467  
e. 0.9896  
f. 0.4348  
g. 0.9818  
h. 0.1435

EXERCISE 2

EXERCISE 3

a. Sin 70° = \( \frac{X}{45} \)
   \[ \therefore X = 45 \times \sin 70° \]
   \[ = 45 \times 0.9397 \]
   \[ = 42.2862 \]
   \[ = 42.29 \]

b. \( \sin 55°55' = \frac{X}{380} \)
   \[ X = 299.10 \]

c. \( \sin 44°5' = \frac{X}{720} \)
   \[ X = 500.91 \]
EXERCISE 4

a. \( \cos 38^\circ = R \)
   \[
   \begin{align*}
   \therefore R &= 95 \\
   &= 95 \times \cos 38^\circ \\
   &= 95 \times 0.7880 \\
   &= 74.8610 \\
   &= 74.86
   \end{align*}
   \]

b. \( \cos 59^\circ12' = R \)
   \[
   \begin{align*}
   R &= 310 \\
   &= 158.73
   \end{align*}
   \]

c. \( \cos 28^\circ30' = R \)
   \[
   \begin{align*}
   R &= 208 \\
   &= 182.79
   \end{align*}
   \]

EXERCISE 5

a. \( \tan 17021' = Z \)
   \[
   \begin{align*}
   \therefore Z &= 290 \\
   &= 290 \times \tan 17^\circ21' \\
   &= 290 \times 0.3124 \\
   &= 90.6026 \\
   &= 90.60
   \end{align*}
   \]

b. \( \tan 81^\circ = Z \)
   \[
   \begin{align*}
   Z &= 67.4 \\
   &= 425.55
   \end{align*}
   \]

EXERCISE 6

a. \( \cos 26^\circ = W \)
   \[
   \begin{align*}
   \therefore W &= 31.5 \\
   &= 31.5 \times \cos 26^\circ \\
   &= 31.5 \times 0.8988 \\
   &= 28.3120 \\
   &= 28.31
   \end{align*}
   \]

b. \( \tan 49^\circ = W \)
   \[
   \begin{align*}
   \therefore W &= 49.21 \\
   &= 56.61
   \end{align*}
   \]

c. \( \sin 22^\circ = W \)
   \[
   \begin{align*}
   W &= 480 \\
   &= 179.81
   \end{align*}
   \]

EXERCISE 7

a. \( \sin 68^\circ5' = 19.21 \)
\[ W = 20.71 \]  

b. \[ \sin 50^\circ 8' = 20.67 \] 
\[ W = 26.93 \]  

c. \[ \tan 29^\circ 10' = 74 \] 
\[ W = 132.59 \]  

d. \[ \sin 37^\circ 49' = 37.36 \] 
\[ W = 60.93 \]  

e. \[ \sin 42^\circ = 198.5 \] 
\[ W = 296.65 \]  

**EXERCISE 8**  

a. 50°24'  
b. 90°  
c. 0  
d. 15°6'  
e. 15°36'  
f. 73°
EXERCISE 9

a. \[ \sin \theta = \frac{4}{9} \]
\[ \sin 26^\circ 23' = \frac{4}{9} \]

b. \[ \cos \theta = \frac{26}{38} = \frac{13}{19} \]
\[ \cos 46^\circ 50' = \frac{13}{19} \]

c. \[ \tan \theta = \frac{40}{13} \]
\[ \tan 72^\circ - \frac{40}{13} \]

d. \[ \cos \theta = \frac{25}{60} = \frac{5}{12} \]
\[ \cos 65^\circ 23' = \frac{5}{12} \]

e. Opposite side \[ \sin \theta = \frac{X}{10} \]
\[ X = 8.09 \text{mm} \]
Adjacent Side \[ \cos \theta = \frac{R}{10} \]
\[ R = 5.88 \text{mm} \]

f. Hypotenuse (Z) \[ \sin \theta = \frac{13}{Z} \]
\[ Z = 31.96 \text{ units} \]
Adjacent (R) \[ \tan \theta = \frac{13}{R} \]
\[ R = 29.20 \text{ units} \]