Electro Critical Skills Resource Suite

Entry Level Literacy and Numeracy Assessment for the Electrotechnology Trades

Enrichment Resource

UNIT 12: Transposition

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managing apprentice progression

An E-Oz Energy Skills Australia project.

ENERGY SKILLS AUSTRALIA

TRANSPOSITION

A large portion of practical mathematics in Electrical studies consists of working with electrical formulas (equations)

These formulae are algebraic expressions showing how some value varies with respect to others.

Often the formula is given in such a way that to use the available data you need to rearrange or transpose the formula. Electrical formulae are used to calculate values such as : circuit impedance, resistance, capacitance, applied voltages, efficiency of transformers and power ratings.

LEARNING OUTCOME

- Can transpose formulae to solve for an unknown value.

PERFORMANCE CRITERIA

- Simplifies and expands algebraic expressions.
- Substitutes values in algebraic expressions to find a solution.
- Understands the rules of transposition.
- Uses transposition to solve equations.
- Transposes electrical formulae to find an unknown value.

PART A

REVIEW OF BASIC ALGEBRA

In order to use and transpose formulae it is necessary to understand how to perform operations using algebra.

Algebra is essentially the mathematics of symbols with the laws of arithmetic being applied to letters instead of numbers. The letters or symbols can stand for a number.

For Example: 7b 2zy 1R

Note: 7b means 7 x b

2zy means 2 x z x y

1R means 1 x R

In algebra the **multiplication** sign is not written.

Grouping Like Terms

When **adding** and **subtracting** in algebra only the terms containing the same symbols can be added or subtracted.

Example 1

2a + 4b + 3a
= (2a + 3a) + 4b
= 5a + 4b

The ‘a’ terms have been added together

Example 2

7r + 3c

This cannot be simplified because the pronumerals (represented by the letters ‘r’ and ‘c’) are different.

It is like trying to add 7 resistors and 3 capacitors. The combined result is still 7 resistors and 3 capacitors.

Example 3

9y-3y = 6y

Example 4

6w - 4z + 10

This cannot be simplified.

EXERCISE 1

Simplify the following where possible:

a) $4A+3A$

b) $3x-y$

c) $8m - 7m$

d) $2p + 9a + 4p - 2a$

e) $6R1 - 4R2 + 3R1$

f) $26w - 26w$

g) $4V + 6W - 2V + 8Z$

Use the answer sheet to check your work.

Example 5 6xy + 5xw

This cannot be simplified

Example 6 2efg + 3efg + 4fhg = 5efg + 4fhg

Example 7 9xy - 4yx

= 9xy - 4xy

= 5xy

Note: xy is the same as yx. The order of the letters does not matter.

EXERCISE 2

a) 4ab+ 6ab

b) xy + 5xy

c) 8pqr - 7pqr

d) 5ef + 2fg - 3ef

e) 15mn -15mn

f) 8wx + 3xw

g) $9R_1 R_2 + 7L_1 L_2 - 5R_1 R_2 + 3L_2 L_1$

h) 2ghi + 3igh + 9hig - 2hji

☑ **Use the answer sheet to check your work.**

Example 8 f x g x h = fgh

Example 9 3z x 4y = 12zy

EXERCISE 3

Rewrite the following. You will need to remove the multiplication sign.

a) p x q x r

b) 3m x n

c) 6h x 5l

d) 2R1 x 4R2

e) 5w x 2z x3y

Expanding the brackets and grouping like terms:

Example 10 2(x - y)= 2 x - 2y

Example 11 7 (a - 2b) = 7 x a - 7 x 2 b

= 7a - 14b

Example 12 4 (2F - G) + 6F = 8F - 4G + 6F

= 14F-4G

Example 13 $p(3m - 3r) - m(2p + 4r)$

$= 3pm - 3pr - 2mp - 4mr$

$= 3pm - 2mp - 3pr - 4mr$

$= pm - 3pr - 4mr$

EXERCISE 4

Simplify the following:

a) $3 (a + b)$

b) $5 (m - n)$

c) $12 (p - 2q)$

d) $4 (2R1 + 3R2) -3R2$

e) $3 (r + s) + 4 (2r - s)$

f) $6 (m + 2n) + 2 (4m + n)$

g) $8 (3s + 2t) + 6 (2s - 3t)$

h) $4 (3x - y) - (x - y)$

(Remember: - x- = +, - x+ =-)

I) $3r (s + 2q) + rq$

☑ **Use the answer sheet to check your work.**

SIMPLIFYING FRACTIONS

Example 14

$$\frac{p}{2} + \frac{4p}{5}$$

(The lowest common denominator is 10)

$$= \frac{5p}{10} + \frac{8p}{10}$$

$$= \frac{13p}{10}$$

Example 15

$$= \frac{M}{4} + \frac{M+2}{3}$$

(The lowest common denominator is 12)

$$= \frac{3M}{12} + \frac{4(M+2)}{12}$$

$$= \frac{3M}{12} + \frac{4M+8}{12}$$

$$= \frac{7M+8}{12}$$

EXERCISE 5

Simplify the following expressions:

a) $\frac{r}{3} + \frac{r}{5}$

b) $\frac{w}{3} + \frac{2w}{9}$

c) $\frac{x+4}{2} + \frac{3x+1}{4}$

d) $\frac{2p+3m}{4} + \frac{2m+p}{3}$

☑ **Use the answer sheet to check your work.**

EXPONENTS (OR POWERS)

In the term $3y^2$:

- 3 is the coefficient
- y is the base
- 2 is the exponent

To simplify the equation $2y^2 + y^2$, the y^2 terms can be added together:

$= 2y^2 + y^2$
$= 3y^2$

Algebraic terms can be added or subtracted as long as their bases and exponents are the same.

Example 16 $3x^2 + 2x^2$

$= 5x^2$

Example 17 $y^3 – 3y^3 + 4y^3 + x$

$= y^3 + 4y^3 – 3y^2 + x$

$= 5y^3 – 3y^2 + x$

EXERCISE 6

Simplify the following:

a) $3m^2 - m^2$

b) $4r^2 - 2r$

c) $R^3 + 2R - P + 2R^3$

d) $3p^2q + 2qp - qp^2$

e) $6x^2y - 12x^3y + yx^2$

f) $\frac{2s^2 - r}{3} + \frac{s^2 + r}{4}$

☑ **Use the answer sheet to check your work.**

SUBSTITUTING IN FORMULAE

Substituting involves giving a letter (pronumeral) in an expression or formulae a number value so that an answer can be found.

Example 1

Find the value of $z + 4$ if $z = 3$

Using substitution $z + 4 = 3 + 4 = 7$

Example 2

To calculate the number of watts in a circuit you can use the formulae:

	watts	=	volts x amps
	W	=	V x A
Or	W	=	VA

Calculate the number of watts if V = 5 and A = 3. Using substitution you get:

W	=	5 x 3
W	=	15

Answer: There are 15 watts in the circuit.

EXERCISE 7

Find the value of the following if W = 2 and X = 4

a) $5 + W$

b) $2W + X$

c) $6(2W - X)$

d) $\frac{X}{W}$

e) $WX-3$

f) $\frac{WX}{100}$

g) $W(2X + 5)$

h) $\frac{XW}{X + 4 + W}$

i) $\frac{3(4W + 2X)}{6W}$

j) $3W^2 - 2X + X^2W$

k) $\frac{X^2}{W}$

EXERCISE 8

a) Ohm's Law

The formula for calculating the voltage drop across a resistance when a current is flowing through it, is given by:

V = IR (answer in volts)

where V = voltage drop (volts)

I = current (amps)

R = resistance (ohms)

Find the voltage drop (V) given the following current and resistance values:

(i) I = 3A R=2Ω

(ii) I = 5A R = 1.2 Ω

(iii) I = 15 R = 0.8 Ω

b) **Electrical Power**

A formula for calculating the value of power drawn from a supply is:

$$P = \frac{V^2}{R}$$

where	P	=	power (watts)
	V	=	voltage drop (volts)
	R	=	resistance (ohms)

Find the value of power given the following voltage drop and resistance values.

(i) V = 240V R = 23Ω

(ii) V = 12V R = 24Ω

(iii) V = 49V R = 200Ω

c) Impedance

The formula for calculating the impedance in a circuit is:

$$Z = \sqrt{R^2 + X^2}$$

where	Z	-	impedance (ohms)
	R	-	resistance (ohms)
	X	-	reactance (ohms)

Find the impedance of a circuit given the following values for the resistance and reactance:

(i) $R = 8\Omega$ $X = 12\Omega$

(ii) $R=3\Omega$ $X = 5.2\Omega$

(iii) $R = 42\Omega$ $X = 56\Omega$

d) Efficiency

The formula used for calculating the efficiency of an electric motor is:

$$\eta = (\frac{Pout}{Pin} x100)\%$$

where η = efficiency (%)

Pout = Power output (watts)

Pin = Power input (watts)

Calculate the efficiency of the electric motors with the following Pout and Pin values:

i) P*out* = 120W P*in* = 160W

ii) P*out* = 135W P*in* = 225W

iii) P*out* = 3000W P*in* = 3357W

☑ **Use the answer sheet to check your work.**

SOLVING EQUATIONS

An equation is a mathematical statement that two expressions are equal to each other e.g. $x + 6 = 8$

To solve an equation we find the number or numbers that make the equation true e.g. find the value of x in the above equation.

There is one basic rule in working with equations – to maintain equality you must do the same thing to each side.

Whatever is done to one side of the equation must also be done to the other side.

Example 1 $x + 6 = 8$

To solve for x it is necessary to isolate x on one side of the equation and the numbers on the other.

Step 1

$x + 6 = 8$ Subtract 6 from both sides

$x + 6 - 6 = 8 - 6$

$\mathbf{x = 2}$

Step 2

To check the answer, substitute 2 for x in the original equation.

$x + 6 = 8$

If $x = 2$

$2 + 6 = 8$

$8 = 8$

Since both sides are equal, the solution must be correct.

Example 2 $t - 3 = 10$

Step 1 Add 3 to both sides $t - 3 + 3 = 10 + 3$

Step 2 $t = 13$

EXERCISE 9

Solve the following equations:

a) $m + 8 = 18$

b) $p - 3 = 9$

c) $c - ½ = 2$

EXERCISE 10

Solve the following equations:

Example 3 $6x = 12$

Divide both sides by 6

$6x \div 6 = 12 \div 6$

$x = 2$

Example 4 $\frac{m}{4} = 3$

Multiply both sides by 4

$\frac{M}{4} \times 4 = 3 \times 4$

$m = 12$

a) $8y = 24$

b) $3r = 123$

c) $\frac{r}{2} = 9$

d) $\frac{y}{7} = 2$

EXERCISE 11

Solve the following equations:

Example 5 $3x + 4 = 19$

Subtract 4 from both sides

$3x + 4 - 4 = 19 - 4$

$3x = 15$

Divide both sides by 3

$3x + 3 \quad = 15 \div 3$

$x = 5$

a) $5w + 2 = 8$

b) $3p - 1 = 11$

c) $5 + 3a = 8$

d) $8t - 4 = 20$

EXERCISE 12

Solve the following equations:

Example 6 $4 (p + 3) = 20$

Divide both sides by 4

$$\frac{4(p+3)}{4} = \frac{20}{4}$$

$p + 3 = 5$

Subtract 3 from both sides

$p + 3 - 3 = 5 -3$

$p =2$

a) $2 (m - 4) = 14$

b) $6 (r + 8) = 120$

c) $9 (y – ½) = 36$

d) $5 (b + 12) = 65$

EXERCISE 13

Solve the following equations:

Example 7

$$\frac{q+3}{2} = 4$$

Multiply both sides by 2

$$\frac{(q+3)}{\cancel{2}} x\cancel{2} = 4x2$$

q + 3 = 8

Subtract 3 from both sides

q + 3 – 3 = 8 - 3

q = 5

Example 8

$\frac{m}{2} - 3 = 4$

Add 3 to both sides

$\frac{m}{2} - 3 + 3 = 4 + 3$

$\frac{m}{2} = 7$

Multiply both sides by 2

$\frac{m}{\cancel{2}} \times \cancel{2} = 7 \times 2$

m = 14

a) $\frac{p+3}{6} = 2$

b) $\frac{x}{5} - 4 = 6$

c) $\frac{r-2}{3} = 3$

d) $\frac{w}{4} - 1 = 8$

EXERCISE 14

Solve the following equations:

Example 9

$$\frac{9y+3}{3}=10$$

Multiply both sides by 3

$$\frac{(9y+3)}{3}x3=10x3$$

Subtract 3 from both sides

9y + 3 = 30
9y + 3 - 3 = 30 - 3
9y = 27 Divide both sides by 9
y = 3

a) $\frac{3m+1}{4}=1$

b) $\frac{2p-3}{3}=7$

c) $\frac{6(r+3)}{5}=6$

d) $\frac{2(q+2)}{4}=8$

EXERCISE 15

Solve the following:

Example 10 $3x - 3 = x + 1$

Subtract x from both sides
$3x - 3 - x = x + 1 - x$

Collect like terms
$2x - 3 = 1$

Add 3 to both sides
$2x - 3 + 3 = 1 + 3$
$2x = 4$

Divide both sides by 2
$x = 2$

Example 11 $4(s + 1) = 3(s + 2)$

Expand the brackets first
$4s + 4 = 3s + 6$

Group the like terms
$4s - 3s + 4 = 3s + 6 - 3s$
$s + 4 = 6$
$s = 6 - 4$
$s = 2$

a) $5p - 7 = 3p + 5$

b) $10m + 6 = 11m - 4$

c) $3(w - 2) = w + 2$

d) $3(4x-5)=2(5x-2)$

☑ **Use the answer sheet to check your work.**

PART B

TRANSPOSING FORMULAE

A formula is often given in such a way that to use the available information you need to rearrange or transpose the formula.

For example, the formula for calculating the number of watts in a circuit is:

$$W=VA$$

ie. **watts = volts x amps**

If you want to find the volts (V) you will need to transpose the formula to make V the subject.

ie. **V = ...**

In many ways transposing a formula is similar to solving an equation, although in this case an exact value for the letter (eg V) is not found.

Remember the rule:

Whatever is done to one side of an equation must also be done to the other side.

This rule for solving equations and the methods used in the previous section apply to **transposing** formulae.

EXERCISE 1

Transpose the following formulae to solve for the indicated letter.

Example 1

$x = w - z$ Solve for w
Add z to both sides
$x + z = w - z + z$
$x + z = w$

Example 2

$A + B = C + D$ Solve for A
Subtract B from both sides
$A + B - B = C + D - B$
$A = C + D - B$

a) $p = q + r$
Solve for r

b) $R_t = R_1 + R_2 + R_3$
Solve for R_2

c) $x = y - w$
Solve for w

d) $f = g + h - i$
Solve for h

e) $P_T = P_1 + P_2 + P_3$
Solve for P_3

EXERCISE 2

Transpose the formulae to solve for the indicated letter.

Example 3 $s = 2r + q$ Solve for q

Subtract 2r from both sides

$s - 2r = 2r + q - 2r$

$s - 2r = q$

Example 4 $l = 3m - n$ Solve for m

Add n to both sides

$l + n = 3m - n + n$

$l + n = 3m$

Divide both sides by 3

$$\frac{l+n}{3} = \frac{3m}{3}$$

$$\frac{l+n}{3} = m$$

a) $a = 4b + c$
Solve for c

b) $z = 2w - x$
Solve for w

c) $s = 2r + t$
Solve for r

d) $f = 3g - h$
Solve for h

e) $L_t = L_1 + L_2 + 2M$
Solve for M

EXERCISE 3

Transpose the following formulae to make the indicated letter the subject.

Example 5 W = VA Solve for V

Divide both sides by A

$$\frac{W}{A} = \frac{V\cancel{A}}{\cancel{A}}$$

$$\frac{W}{A} = X$$

Example 6 x = 6yw Solve for y

Divide both sides by 6w

x ÷ 6w = 6yw ÷ 6w

$$\frac{x}{6w} = \frac{6yw}{6w}$$

$$\frac{x}{6w} = y$$

a) Q = VC

Solve for C

b) ab = cd

Solve for d

c) k = 9lm

Solve for l

d) $X_L = 2\eta f L$

Solve for L

e) $P = 1^2R$

Solve for R

EXERCISE 4

Transpose the following formulae to solve for the indicated letter.

Example 7

m $= \frac{2k}{n}$ Solve for k

Multiply both sides by n

mn = $\frac{2k}{\cancel{n}} \times \cancel{n}$

mn = 2k

Divide both sides by 2

$\frac{mn}{2} = k$

Example 8

R = $\frac{PL}{A}$ Solve for A

Multiply both sides by A

R x A = $\frac{PL}{\cancel{A}} \times \cancel{A}$

RA = PL

Divide both sides by R

$\frac{\cancel{RA}}{\cancel{R}} = \frac{PL}{R}$

A = $\frac{PL}{R}$

a) $a = \frac{bc}{d}$

Solve for c

b) $f = \frac{np}{120}$

Solve for n

c) $V_e = \frac{1000C_d}{Ll}$

Solve for L

d) $P.f = \frac{R}{Z}$

Solve for f

EXERCISE 5

Transpose the following formulae to make the indicated letter the subject.

Example 9

$c = \frac{3(d - e}{3}$ solve for d

Divide both sides by 3

$\frac{c}{3} = \frac{\cancel{3}(d - e)}{\cancel{3}}$

$\frac{c}{3} = d - e$

Add e to both sides

$\frac{c}{3} + e = d - e + e$

$\frac{c}{3} + e = d$

Example 10

y = x (vw + 2) Solve for w

Divide both sides by x

$\frac{y}{x} = \frac{\cancel{x}(vw + 2}{\cancel{x}}$

$\frac{y}{x} = vw + 2$

Subtract 2 from both sides

$\frac{y}{x}$ - 2 = vw + 2 - 2

$\frac{y}{x}$ - 2 = vw

Divide both sides by v

$\frac{y - 2x}{x} \div v = vw \div v$

$\frac{y - 2x}{xv} = \frac{\cancel{v}w}{\cancel{v}}$

$\frac{y - 2x}{xv} = w$

a) m = 2 (ln + 1)
Solve for n

b) a = bc (3d + e)
Solve for d

c) x = y (6w - z)
Solve for w

d) p=q(rs –2)
Solve for s

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EXERCISE 6

Transpose the following formulae to solve for the indicated letter.

Example 11

x = $2y^2$ Solve for y

Divide both sides by 2

$$\frac{x}{2} = y^2$$

Find the square root of both sides

$$\sqrt{\frac{x}{2}} = \sqrt{y^2}$$

$$\sqrt{\frac{x}{2}} = y$$

Example 12

P = I^2R Solve for I

Divide both sides by R

$$\frac{P}{R} = \frac{I^2R}{R}$$

Find the square root of both sides

$$\sqrt{\frac{P}{R}} = \sqrt{I^2}$$

$$\sqrt{\frac{P}{R}} = I$$

Example 13

Q = $\sqrt{S^2 - P^2}$ Solve for S^2

Square both sides

Q = $\sqrt{S^2 - P^2}^{\,2}$

Q=S-P

Add P to both sides

$Q^2 + P^2 = S^2 - P^2 + P^2$

$Q^2 + P^2 = S^2$

a) $a = b^2$
Solve for b

b) $y = 7x^2 + 3$
Solve for x

c) $x = \sqrt{\frac{y}{w}}$
Solve for w

d) $F_r = \sqrt{F_1^{\ 2} + F_2^{\ 2}}$
Solve for F_2

e) $E = ½\ mv^2$
Solve for V

☑ **Use the answer sheet to check your work.**

TRANSPOSING ELECTRICAL FORMULAE

EXERCISE 7

a) Ohm's Law

Ohm's law states that in any electrical circuit the current is directly proportional to the voltage and inversely proportional to the circuit resistance.

The formula for this relationship is:

$$I = \frac{V}{R}$$

Where:	I	=	current (amps)
	V	=	potential difference (volts)
	R	=	resistance (ohms)

i) What is the value of I if the potential difference is 6 volts and the resistance is 2 ohms?

ii) Transpose the formula to make V the subject and hence calculate the voltage (V) if I = 8 amps and R = 12Ω.

iii) Transpose the formula to make R the subject and hence calculate the resistance (R) if V = 11.5 volts and I = 2 amps.

b) Electrical Power

A formula for calculating the value of power in an electrical circuit is:

	P	=	I^2R
Where:	P	=	power (watts)
	I	=	current (amps)
	R	=	resistance (ohms)

i) Calculate the power in a circuit if I = 0.5 amps and R = 24 ohms.

ii) Transpose the formula to make resistance the subject and hence calculate the value of R if P = 1.5 watts and I = 0.5 amps.

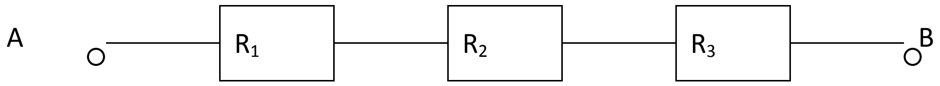
iii) Transpose the formula to make current the subject and hence calculate the value of I if R = 18 ohms and P = 4.5 watts.

c) Resistance of a Series Circuit

To find the total resistance to current flow in any series connected circuit, the values in ohms of the individual resistors are added. The formula for calculating this total resistance to current flow is:

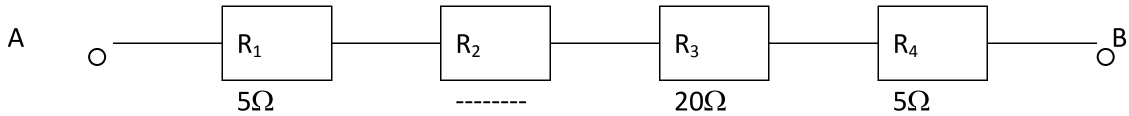
$R_{total} = R_1 + R_2 + R_3 + R_4$ .....

E.g. The diagram below represents 3 resistors in a circuit between terminal A and B.



$\therefore\ R_{total} = R_1 + R_2 + R_3$

i) Calculate the total resistance (R_{total}) in a series circuit with 3 resistors where $R_1 = 10\Omega$, $R_2 = 10\Omega$, and $R_3 = 5\Omega$.



ii) Find the value of R_2 in the above circuit if $R_{total} = 40\Omega$.

iii) Find the value of R_4 in a circuit with 4 resistors if $R_1 = 12\Omega$, $R_2 = 8.2\Omega$, $R_3 = 6.8\Omega$, and the total resistance is 66.

d) A.C. Circuits

Pythagoras' Theorem is used to calculate the resistance, reactance and impedance in AC circuits. The formula used is:

$Z^2 = R^2 + X^2$

or: $Z = \sqrt{R^2 + X^2}$

Where:

Z = impedance (ohms)
X = reactance (ohms)
R = resistance (ohms)

i) What is the impedance of an AC circuit with a reactance (X) of 3 ohms and a resistance (R) of 5 ohms?

ii) Transpose the formula to make R the subject and hence calculate the value of R if X = 12 ohms and Z = 24 ohms.

iii) What is the reactance of an AC circuit with an impedance of 20 ohms and a resistance of 8 ohms? (Remember. you will need to transpose the formula to make the reactance the subject.)

e) The rate of doing work (power) for a rotating body is found by using the formula:

$$P = 2\prod nT$$

Where:	P	=	power in watts
	n	=	revolutions per second (r/s)
	T	=	Torque in newton-metres (Nm)
	$2\prod$	=	6.28

i) Find the power (P) of an electric motor which is operating at 35 r/sec and has a torque of 20 Nm.

ii) Transpose the formula to make torque (T) the subject. Calculate the value of T for a compressor if the revolutions per second (n) is 20 r/s and the power (P) is 800 watts.

iii) What is the rotational speed (n) of an electric motor if the torque is 6 Nm and the power is 748 watts?

☑ **Use the answer sheet to check your work.**

ANSWERS

PART A

EXERCISE 1

a) $4A + 3A = 7A$ '
b) $3x - y$
c) $8m - 7m = m$
d) $2p + 9a + 4p - 2a = 6p + 7a$
e) $6R_1 - 4R_2 + 3R_1 = 9R_1 - 4R_2$
f) $26w - 26w = 0$
g) $4V + 6W - 2V + 8Z = 2V + 6W + 8Z$

EXERCISE 2

a) $4ab + 6ab = 10ab$
b) $xy + 5xy = 6xy$
c) $8pqr - 7pqr = pqr$
d) $5ef + 2fg - 3ef = 2ef + 2fg$
e) $15mn - 15mn = 0$
f) $8wx + 3xw = 8wx + 3wx = 11wx$
g) $9 R_1 R_2 + 7L_1L_2 - 5 R_1 R_2 + 3L_1L_2 = 4 R_1 R_2 + 10L_1L_2$
h) $2ghi + 3igh + 9hig - 2hji = 14ghi - 2hji$

EXERCISE 3

a) $p \times q \times r = pqr$
b) $3m \times n = 3mn$
c) $6h \times 51 = 30hl$
d) $2R_1 \times 4R_2 = 8R_1 R_2$
e) $5w \times 2z \times 3y = 30wzy$,

EXERCISE 4

a) $3(a + b) = 3a + 3b$
b) $5(m - n) = 5m - 5n$
c) $12(p - 2q) = 12p - 24q$
d) $4(2R_1 + 3R_2) - 3R_2$
$= 8R_1 + 12R_2 - 3R_2$
$= 8R_1 + 9R_2$
e) $3(r + s) + 4(2r - s)$
$= 3r + 3s + 8r - 4s$
$= 11r - s$
f) $6(m + 2n) + 2(4m + n)$
$= 6m + 12n + 8m + 2n$
$= 14m + 14n$

g) $8(3s + 2t) + 6(2s - 3t)$

$= 24s + 16t + 12s - 18t$

$= 36s - 2t$

h) $4(3x - y) - (x - y)$

$= 12x - 4y - x + y$

$= 11x - 3y$

i) $3r(s + 2q) + rq$

$= 3rs + 6rq + rq$

$= 3rs + 7rq$

EXERCISE 5

a) $\frac{r}{3} + \frac{r}{5}$

$= \frac{5r}{15} + \frac{3r}{15}$

$= \frac{8r}{15}$

b) $\frac{w}{3} + \frac{2w}{9}$

$= \frac{3w}{9} + \frac{2w}{9}$

$= \frac{5w}{9}$

c) $\frac{x + 4}{2} + \frac{3x + 1}{4}$

$= \frac{2(x + 4)}{4} + \frac{3x + 1}{4}$

$= \frac{2x + 8 + 3x + 1}{4}$

$= \frac{5x + 9}{4}$

d) $\frac{2p + 3m}{4} + \frac{2m + p}{3}$

$= \frac{3(2p + 3m)}{12} + \frac{4(2m + p)}{12}$

$= \frac{6p + 9m}{12} + \frac{8m + 4p}{12}$

$= \frac{10p + 17m}{12}$

EXERCISE 6

a) $3m^2 - m^2 = 2m^2$

b) $4r^2 - 2r$

c) $R^3 + 2R - P + 2R^3$

$= 3R^3 + 2R - P$

d) $3p^2q + 2qp - qp^2$

$= 2p^2q + 2qp$

e) $6x^2y - 12x^3y + yx^2$

$= 7x^2y - 12x^3y$

f) $\frac{2s^2 - r}{3} + \frac{s^2 + r}{4}$

$= \frac{4(2s^2 - r)}{12} + \frac{3(s^2 + r)}{12}$

$= \frac{8s^2 - 4r}{12} + \frac{3s^2 + 3r}{12}$

$= \frac{11s^2 - r}{12}$

EXERCISE 7

a) $5 + W = 5 + 2 = 7$

b) $2W + X$

$= 4 + 4$

$= 8$

c) $6(2W - X)$

$= 6(4 - 4)$

$= 0$

d) $X = 4 = 2$

$W = 2$

e) $WX - 3 = 8 - 3 = 5$

f) $\frac{WX}{100} = \frac{8}{100} =..\frac{2}{25}$

g) $W(2X + 5) = 2(8 + 5)$

$= 26$

h) $\frac{XW}{X+4+W} = \frac{8}{10} = \frac{4}{5}$

i) $\frac{3(4W + 2X)}{6W} = \frac{3(8 + 8)}{12} = \frac{48}{12} = 4$

j) $3W^2 - 2X + X^2W = 3 \times 4 + 2 \times 4 + 4 \times 4 \times 2$

$= 12 - 8 + 32$

$= 36$

k) $\frac{X^2}{W} = \frac{16}{2} = 8$

EXERCISE 8

a) (i) $V = IR$

$= 3 \times 2$

$\therefore V = 6$ volts

(ii) $V = IR$

$= 5 \times 1.2$

$\therefore V = 6$ volts

(iii) $V = IR$

$= 15 \times 0.8$

$\therefore V = 12$ volts

b) (i) $P = \frac{V^2}{R}$

$= \frac{(240)^2}{23}$

$\therefore P = 2504W$

(ii) $P = \frac{V^2}{R}$

$= \frac{12^2}{24}$

$\therefore P = 6W$

(iii) $P = \frac{V^2}{R}$

$= \frac{(49)^2}{200}$

$= \frac{2401}{200}$

$\therefore P = 12W$

c) (i) $Z = \sqrt{R^2 + X^2}$

$= \sqrt{8^2 + 12^2}$

$\therefore Z = \sqrt{208\Omega}$

$= 14.4\Omega$

(ii) $Z = \sqrt{R^2 + X^2}$

$= \sqrt{3^2 + (5.2)^2}$

$\therefore Z = \sqrt{36.04\Omega}$

$= 6.00\Omega$

(iii) $Z = \sqrt{R^2 + X^2}$

$= \sqrt{42^2 + 56^2}$

$\therefore Z = \sqrt{4900\Omega}$

$= 70\Omega$

d) (i)

$$\eta = \left(\frac{Pout}{Pin} x100 \right)\%$$
$$= \frac{120}{160} x100$$
$$\therefore \eta = 75\%$$

(ii)

$$\eta = \left(\frac{Pout}{Pin} x100 \right)\%$$
$$= \frac{135}{225} x100$$
$$\therefore \eta = 60\%$$

(iii)

$$\eta = \left(\frac{Pout}{Pin} x100 \right)\%$$
$$= \frac{3000}{3357} x100$$
$$\therefore \eta = 89.37\%$$

EXERCISE 9

a) m + 8 = 18
m + 8 – 8 = 18 - 8
m = 10

b) p - 3 = 9
p - 3 + 3 = 9 + 3
p = 12

c) ½ = 2
c = 2 ½

EXERCISE 10

a) 8y = 24
y = 24 ÷ 8
y = 3

b) 3r = 123
r = 41

c) $\frac{r}{2} = 9$
r = 9 x 2
r = 18

d) $\frac{y}{7} = 2$
y = 14

energise oz
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EXERCISE 11

a) $5w + 2 = 8$

$5w = 6$

$w = \frac{6}{5}$

b) $3p - 1 = 11$

$3p = 12$

$p = 4$

c) $5 + 3A = 8$

$3A = 3$

$A = 1$

d) $8t - 4 = 20$

$8t = 24$

$t = 3$

EXERCISE 12

a) $2(m - 4) = 14$

$\frac{2(m - 4)}{2} = \frac{14}{2}$

$m - 4 = 7$

$m = 11$

b) $6(r + 8) = 120$

$\frac{6(r + 8)}{6} = \frac{120}{6}$

$r + 8 = 20$

$r = 12$

c) $9(y - ½) = 36$

$\frac{9(y - ½)}{9} = \frac{36}{9}$

$y - ½ = 4$

$y - ½ + ½ = 4$

$y = 4 ½$

d) $5(b + 12) = 65$

$\frac{5(b + 12)}{5} = \frac{65}{5}$

$b + 12 = 13$

$b = 1$

EXERCISE 13

a) $\frac{p + 3}{6} = 2$

$p + 3 = 12$

$p = 9$

b) $\frac{x}{5} - 4 = 6$

$\frac{x}{5} - 4 + 4 = 6 + 4$

$\frac{x}{5} = 10$

$\frac{x}{5} \times 5 = 10 \times 5$

$x = 50$

c) $\frac{r - 2}{3} = 3$

$\frac{r - 2}{3} \times 3 = 3 \times 3$

$r - 2 = 9$

$r = 11$

d) $\frac{w}{4} - 1 = 8$

$\frac{w}{4} - 1 + 1 = 8 + 1$

$\frac{w}{4} = 9$

$\frac{w}{4} \times 4 = 9 \times 4$

$w = 36$

EXERCISE 14

a) $\frac{3m + 1}{4} = 1$

$\frac{3m + 1}{4} \times 4 = 1 \times 4$.

$3m + 1 = 4$

$3m + 1 - 1 = 4 - 1$

$3m = 3$

$m = 1$

b) $\frac{2p - 3}{3} = 7$

$\frac{2p - 3}{3} \times 3 = 7 \times 3$

$2p - 3 = 21$

$2p - 3 + 3 = 21 + 3$

$2p = 24$

$p = 12$

c) $\frac{6(r + 3)}{5} = 6$

$\frac{6(r + 3)}{5} \times 5 = 6 \times 5$

$6\ (r+3) = 30$

$\frac{6(r + 3)}{6} = \frac{30}{6}$

$r + 3 = 5$

$r = 2$

d) $\frac{2(q+2)}{4} = 8$

$\frac{2(q + 2)}{4} \times 4 = 8 \times 4$

$2\ (q+2) = 32$

$\frac{2\ (q+2)}{2} = 32$

$q + 2 = 16$

$q = 14$

EXERCISE 15

a) $5p - 7 = 3p + 5$

$5p - 7 - 3p = 3p + 5 - 3p$

$2p - 7 = 5$

$2p - 7 + 7 = 5 + 7$

$2p = 12$

$p = 6$

b) $10m + 6 = 11\ m - 4$

$10m + 6 - 10m = 11\ m - 4 - 10m$

$6 = m - 4$

$m - 4 = 6$

$m - 4 + 4 = 6 + 4$

$m = 10$

c) $3(w - 2) = w + 2$

$3w - 6 = w + 2$

$3w - 6 - w = w + 2 - w$

$w - 6 = 2$

$2w - 6 + 6 = 2 + 6$

$2\ w = 8$

$w = 4$

d) $3(4x - 5) = 2(5x - 2)$

$12x - 15 = 10x - 4$

$12x - 15 - 10x = 10x - 4 - 10x$

$2x - 15 = -4$

$2x - 15 + 15 = -4 + 15$

$2\ x = 11$

$x = 5.5$

ANSWERS

PART B

EXERCISE 1

a) $p = q + r$

$p - q = q + r - q$

$p - q = r$

b) $R_t = R_1 + R_2 + R_3$

$R_t - R_1 - R_3 = R_1 + R_2 + R_3 - R_1 - R_3$

$R_t - R_1 - R_3 = R_2$

c) $x = y - w$

$x + w = y - w + w$

$x + w = y$

$x + w - x = y - x$

$w = y - x$

d) $f = 9 + h - i$

$f - g + I = g + h - I - g + i$

$f - g + I = h$

e) $P_T = P_1 + P_2 + P_3$

$P_T - P_1 - P_2 = P_1 + P_2 + P_3 - P_1 - P_2$

$P_T - P_1 - P_2 = P_3$

EXERCISE 2

a) $a = 4b + c$

$a - 4b = 4b + c - 4b$

$a - 4b = c$

b) $z = 2\,w - x$

$z + x = 2\,w - x + x$

$z + x = 2\,w$

$\frac{z + x}{2} = \frac{\cancel{2}\,w}{\cancel{2}}$

$\frac{z + x}{2} = w$

c) $s = 2r + t$

$s - t = 2r + t - t$

$s - t = 2r$

$\frac{s - t}{2} = \frac{2r}{2}$

$\frac{s - t}{2} = r$

d) $f = 3g - h$

$f + h = 3g - h + h$

$f + h = 3g$

$f + h - f = 3g - f$

$h = 3g - f$

e)

$$L_t = L_1 + L_2 + 2M$$

$$L_t - L_1 - L_2 = L_1 + L_2 + 2M - L_1 - L_2$$

$$L_t - L_1 - L_2 = 2M$$

$$\frac{L_t - L_1 - L_2}{2} = \frac{\cancel{2}M}{\cancel{1}}$$

$$\frac{L_t - L_1 - L_2}{2} = M$$

EXERCISE 3

a)

$$Q = VC$$

$$\frac{Q}{V} = \frac{\cancel{V}C}{\cancel{V}}$$

$$\frac{Q}{V} = C$$

b)

$$ab = cd$$

$$\frac{ab}{c} = \frac{\cancel{c}d}{\cancel{c}}$$

$$\frac{ab}{c} = d$$

c)

$$k = 9lm$$

$$\frac{k}{9m} = \frac{9lm}{9m}$$

$$\frac{k}{9m} = 1$$

d)

$$X_L = 2\Pi f_L$$

$$\frac{X_L}{2\Pi f} = \frac{\cancel{2\Pi f}L}{\cancel{2\Pi f}}$$

$$\frac{X_L}{2\Pi f} = L$$

e)

$$\frac{P}{R} = I^2$$

$$\frac{P}{I^2} = \frac{I^2R}{I^2}$$

$$\frac{P}{I^2} = R$$

EXERCISE 4

a)

$$a = \frac{bc}{d}$$

$$\frac{ad}{b} = \frac{bc}{\cancel{d}} \times \cancel{d}$$

$$\frac{ad}{b} = \frac{\cancel{b}c}{\cancel{b}}$$

$$\frac{ad}{b} = c$$

b) $f = \frac{np}{120}$

$120\text{-}f = \frac{np}{\cancel{120}} \times \cancel{120}$

$\frac{120f}{p} = \frac{n\cancel{p}}{\cancel{p}'}$

$\frac{120f}{p} = n$

e) $V_c = \frac{1000V_d}{Ll}$

$V_cLl = \frac{1000V_d}{Ll} \times Ll$

$\frac{V_cLl}{V_cl} = \frac{1000V_d}{V_cl}$

$L = \frac{1000V_d}{V_cl}$

d) $P.f = \frac{R}{2}$

$\frac{P.f}{P} = \frac{R}{2} \times \frac{1}{P}$

$f = \frac{R}{2P}$

EXERCISE 5

a) $m = 2(ln + 1)$

$m = 2ln + 2$

$m\text{-}2 = 2ln + 2 - 2$

$m\text{-}2 = 2ln$

$\frac{m-2}{21} = \frac{2ln}{2l}$

=> $n = \frac{m-2}{2l}$

b) $a = bc\,(3d + e)$

$a = 3bcd + bce$

$a - bce = 3bcd + bce - bce$

$a - bce = 3bcd$

$\frac{a-bce}{3bc} = \frac{\cancel{3bcd}}{\cancel{3bc}}$

=> $d = \frac{a-bce}{3bc}$ or $\frac{a-bce}{3bc} = d$

c) $x = y\,(6w - z)$

$\frac{x}{y} = 6w – z$

$\frac{x + zy}{y} = 6w$

=> $w = \frac{x + yz}{6y}$

d) $p = q\,(rs - 2)$

$p = qrs - 2q$

$p + 2q = qrs$

$\dfrac{p + 2q}{qr} = \dfrac{qrs}{qr}$

⇨ $s = \dfrac{p + 2q}{qr}$ or $\dfrac{p + 2q}{qr} = s$

EXERCISE 6

a) $a = b^2$

$$\sqrt{a} = \sqrt{b^2}$$

$$\sqrt{a} = b$$

b) $y = 7x^2 + 3$

$y - 3 = 7x^2 + 3 - 3$

$y - 3 = 7x^2$

$\dfrac{y - 3}{7} = \dfrac{7x^2}{7}$

$y - 3 = x^2$

$$\sqrt{\frac{y-3}{y}} = \sqrt{x^2}$$

$$\sqrt{\frac{y-3}{7}} = x$$

c) $x = \sqrt{\dfrac{y}{w}}$

$x^2 = \left(\sqrt{\dfrac{y}{w}}\right)^2$

$x^2 = \dfrac{y}{w}$

$wx^2 = \dfrac{y}{\cancel{w}} \times \cancel{w}$

$wx^2 = y$

$$\frac{wx^2}{x^2} = \frac{y}{x^2}$$

$$w = \frac{y}{x^2}$$

d)

$$F_R = \sqrt{F_1^2 + F_2^2}$$
$$F_R^2 = \left(\sqrt{F_1^2 + F_2^2}\right)^2$$
$$F_R^2 = F_1^2 + F_2^2$$
$$F_R^2 - F_1^2 = F_1^2 + F_2^2 - F_1^2$$
$$F_R^2 - F_1^2 = F_2^2$$
$$\sqrt{F_R^2 - F_1^2} = \sqrt{F_2^2}$$
$$\sqrt{F_R^2 - F_1^2} = F_2$$

e)

$E = ½ mv^2$

$E \times 2 = \frac{mv^2}{2} \times 2$

$2E = mv^2$

$\frac{2E}{m} = \frac{mv^2}{m}$

$\frac{2E}{m} = v^2$

$$v = \sqrt{\frac{2e}{m}}$$

EXERCISE 7

a) i)

$I = \frac{V}{R}$

if I = 6 and R = 2

$I = \frac{6}{2}$

I = 3 amps

ii) $I = \frac{V}{R}$

$IR = \frac{V}{\cancel{R}} \times \cancel{R}^1$

IR = V

if I = 8 and R = 12

V = 8 x 12

= 96 volts

iii)

$I = \frac{V}{R}$

$IR = \frac{V}{R} \times X$

$\frac{IR}{I} = \frac{V}{I}$

$R = \frac{V}{I}$

If V = 11.5 and I = 2

$R = \frac{11.5}{2}$

$R = 5.75\Omega$

b) i) $P = 1^2R$

if I = 0.5 and R = 24

$P = (0.5)^2 \times 24$

P = 6 watts

$P = I^2R$

ii) $\frac{P}{I^2} = \frac{I^2R}{I^2}$

$\frac{P}{I^2} = R$

If P = 1.5 and I = 0.15

$R = \frac{1.5}{(0.5)^2}$

$R = 6\Omega$

$P = I^2R$

iii) $\frac{P}{R} = \frac{I^2R}{R}$

$\frac{P}{R} = I^2$

$$\sqrt{\frac{P}{R}} = \sqrt{I^2}$$

$$\sqrt{\frac{P}{R}} = I$$

If R = 18 and P = 4.5

$$I = \sqrt{\frac{4.5}{18}} = \sqrt{0.25}$$

I = 0.5amps

c) i) $R_{total} = R_1 + R_2 + R_3$

$R_{total} = 10 + 10 + 5$

$R_{total} = 25\Omega$

ii) $R_{total} = R_1 + R_2 + R_3 + R_4$

$R_{total} - R_1 - R_3 - R_4 = R_2$

If $R_{total} = 40$, $R_1 = 5$, $R_3 = 20$, $R_4 = 5$

$40 - 5 - 20 - 5 = R_2$

$R_2 = 10\Omega$

iii) $R_{total} = R_1 + R_2 + R_3 + R_4$

$R_{total} - R_1 - R_2 - R_3 = R_4$

If $R_1 = 12$, $R_2 = 8.2$, $R_3 = 6.8$, $R_{total} = 66$

$R_4 = 66 - 12 - 8.2 - 6.8$

$R_4 = 39\Omega$

d) i)

$$Z = \sqrt{R^2 + X^2}$$
$$If X = 3 and R = 5$$
$$Z = \sqrt{5^2 + 3^2}$$
$$= \sqrt{25 + 9}$$
$$= \sqrt{34}$$
$$= 5.83\Omega$$

ii)

$$Z = \sqrt{R^2 + X^2}$$
$$Z^2 = \left(\sqrt{R^2 + X^2}\right)^2$$
$$Z^2 = R^2 + X^2$$
$$Z^2 - X^2 = R^2 + X^2 - X^2$$
$$Z^2 - X^2 = R^2$$
$$\sqrt{Z^2 - X^2} = \sqrt{R^2}$$
$$\sqrt{Z^2 - X^2} = R$$
$$If X = 12 and Z = 24$$
$$R = \sqrt{24^2 - 12^2}$$
$$R = \sqrt{576 - 144}$$
$$= \sqrt{432}$$
$$R = 20.78\Omega$$

iii)

$$Z = \sqrt{R^2 + X^2}$$
$$Z^2 = \left(\sqrt{R^2 + X^2}\right)^2$$
$$Z^2 = R^2 + X^2$$
$$Z^2 - R^2 = R^2 + X^2 - R^2$$
$$Z^2 - R^2 = X^2$$
$$\sqrt{Z^2 - R^2} = \sqrt{X^2}$$
$$\sqrt{Z^2 - R^2} = X$$
$$If Z = 20 and R = 8$$
$$X = \sqrt{20^2 - 8^2}$$
$$= \sqrt{400 - 64}$$
$$= \sqrt{336}$$
$$X = 18.33\Omega$$

e) i) P = 2∏ nT

P = 6.28 x 35 x 20

P = 4396 watts

ii) $P = 2\Pi nT$

$$\frac{P}{2\Pi T} = \frac{2\Pi nT}{2\Pi n}$$

$$\frac{P}{2\Pi T} = T$$

$$if n = 20 and P = 800$$

$$T = \frac{800}{2\Pi x 20}$$

$$= \frac{800}{6.28x20}$$

$$T = 6.37Nm$$

iii) $P = 2\Pi nT$

$$\frac{P}{2\Pi T} = \frac{2\Pi nT}{2\Pi T}$$

$$\frac{P}{2\Pi T} = n$$

$$If T = 6 and P = 748$$

$$n = \frac{748}{2\Pi x6}$$

$$= \frac{748}{6.28x6}$$

$$n = 19.85r / s$$